The Calculation of G_{rms}

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The metric of G_{rms} is typically used to specify and compare the energy in repetitive shock vibration systems. However, the method of arriving at the G_{rms} measurement (input signal filtering, cutoff frequency of the measurement) can have a dramatic effect on the value. It is important to understand how the measurement is made, and to understand its limitations, in order to use it effectively. This paper will describe the metric of G_{rms}, how it is calculated in both the frequency and time domains and what factors can cause variations in G_{rms} calculations.

What is G_{rms}?

Repetitive shock (RS) vibration systems produce a continuously varying pseudo-random broad spectrum vibration. A typical real time signal from an accelerometer mounted on an RS table is shown in figure 1. The root mean square (rms) value of this signal can be calculated by squaring the magnitude of the signal at every point, finding the average (mean) value of the squared magnitude, then taking the square root of the average value. The resulting number is the G_{rms} metric. (Note: Since this paper addresses G_{rms} calculations specifically, all of the discussion here assumes a signal source that is representative of g's (acceleration). However, the discussion would apply equally well to any measured signal.)

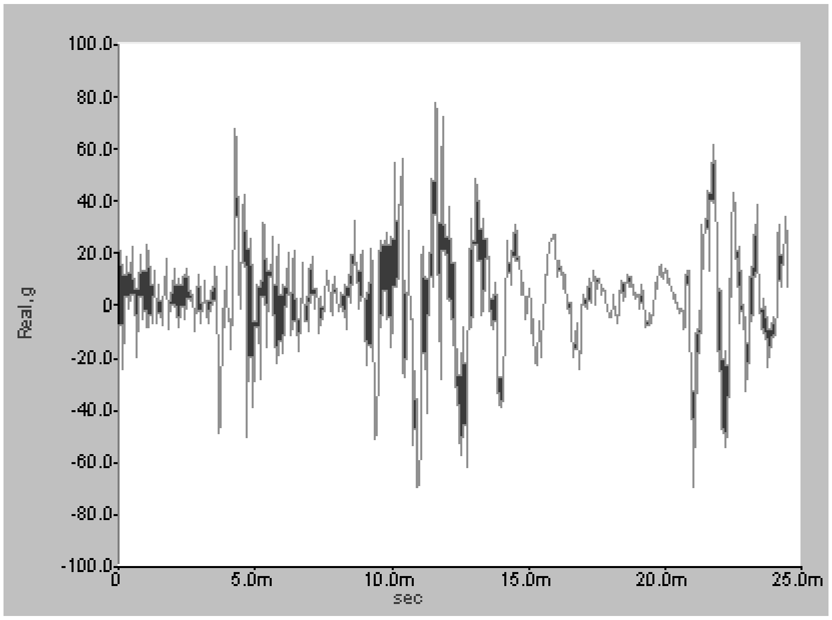


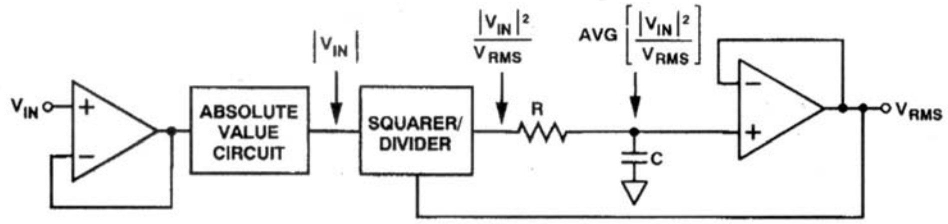
Figure 1. Typical acceleration signal from an RS vibration system

Consider for a moment what the case would be if the signal from the accelerometer on the table were a sine wave rather than the complex signal you see in figure 1. The rms value of a sine wave is easy to calculate from the real time signal – it is simply the peak value of the sine wave times the square root of two. The resultant number would be the G_{rms} value for that vibration level.

Similarly, the rms value of the signal shown in figure 1 can be calculated. However, since this irregular signal is not described by a straight-forward equation, it is not possible to directly calculate the rms value like you can for a sine wave. Fortunately, there are other methods for calculating the rms value of the signal.

Calculating G_{rms} from a real time signal

If you need to continuously measure the rms value of a signal in a system – for example, to control the vibration level of a repetitive shock vibration system – one straightforward method is to employ analog rms converter circuitry[1]. "rms to DC converter" chips are available commercially that produce an output voltage proportional to the rms value of the input signal (figure 2). Adding an input filter to the circuit will limit the frequency range of the signal that is being measured. A microprocessor based control circuit could easily monitor the output rms signal, display a G_{rms} value and control the vibration system to maintain a G_{rms} setpoint.



Implicit Method of RMS-to-DC Conversion Used in AD536A, AD636, AD637, AD736, and AD737

Figure 2. Block diagram of rms converter chip
(Source: Analog Devices data sheet, AD736)

Another method is to perform the rms calculation digitally. This is done by first sampling the analog input signal, yielding a sequence of numbers corresponding to the magnitude of the input signal at each sampled point. The rms calculation now is fairly straightforward. First, each value is squared. Then, all the values are averaged together. A final square root calculation yields the rms number. This method requires the use of the appropriate anti-aliasing filters and sampling circuitry to insure an accurate measurement of the signal across the frequency range of interest.

Calculating G_{rms} from a frequency domain signal

Even though the G_{rms} signal can be easily described as a time domain measurement, it is typically thought of as a frequency domain measurement taken from the Power

Spectrum, or Power Spectral Density, curve. A brief review of the basics of Fourier theory will make this method of determining G_{rms} clearer.

When G_{rms} is calculated using Power Spectrum information it is often thought of as the area under the curve of the Power Spectrum display. More accurately, it is the square root of the integral of the Power Spectrum[2]. This calculation results in the same G_{rms} value as obtained through the time domain measurements thanks to Parseval's Theorem.

Parseval's Theorem (figure 3) states that the energy of a signal is the same whether calculated in the time domain or the frequency domain[3]. Since the Power Spectrum display is in units of G^2, the integral of the Power Spectrum, or the area under the curve, satisfies the right side of Parseval's Theorem, while the summation of the squared values of the digitally sampled time domain signal satisfy the left side of the equation. Taking the square root of each side results in equivalent G_{rms} calculations.

$$\int_{-\infty}^{\infty} h^2(t)dt = \int_{-\infty}^{\infty} |H(f)|^2 df$$

Figure 3. Parseval's Theorem

When you look at the Power Spectrum of a typical vibration signal (figure 4) one thing that can be confusing is the units of the Y axis. For a Power Spectrum, the units are shown as $g^{2/}$Hz, or often $G_{rms}{}^2$/Hz. In this case, the "rms" is not referencing the rms calculation in the time domain described above. Rather, it is an indication of the measurement used for the sinusoidal components represented in the Fourier Transform.

The Fourier Transform of a signal shows the frequency and amplitude of the sine waves that, when summed, would form the time domain signal. If the amplitude of these sine waves is measured as an rms value, then the resultant Y axis units for the Power Spectrum in the frequency domain is $G_{rms}{}^2$/Hz. Indeed, the definition of the Power Spectrum requires that the units be in this form.[2] While some spectrum analyzers will allow choices of Y axis units that include $G_{rms}{}^2$/Hz, $g_{peak}{}^2$/Hz, etc., the only units that result in a Power Spectrum (and hence that can be used to directly calculate G_{rms} as described above) are $G_{rms}{}^2$/Hz.

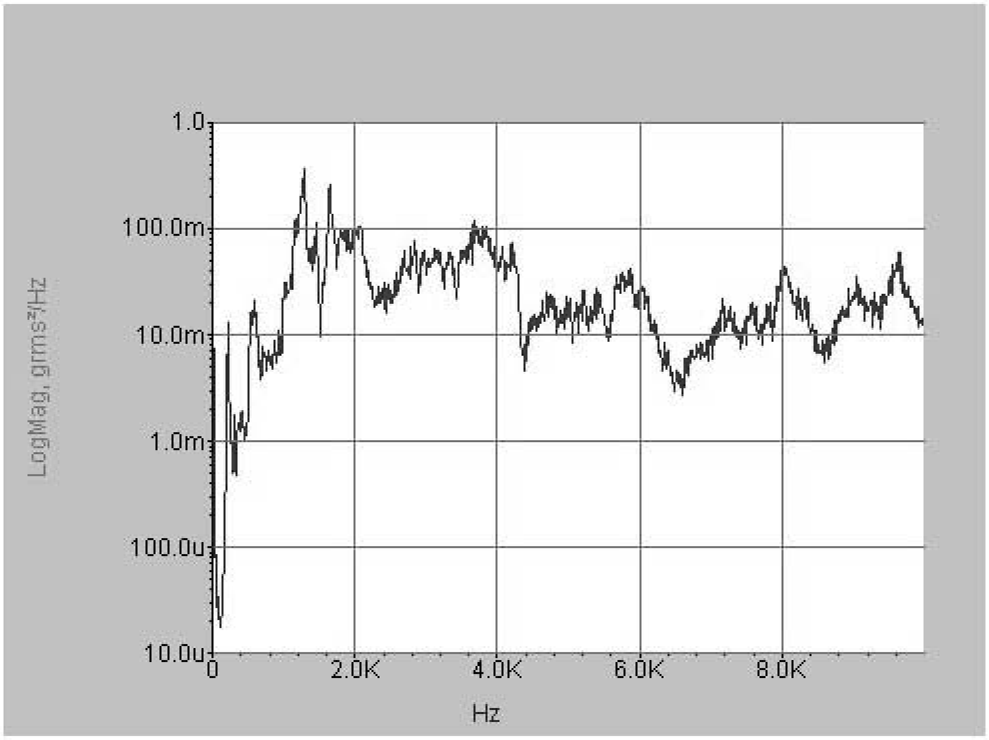


Figure 4. Power Spectrum of an RS vibration system

Variations that affect G_{rms} calculations

If you use the G_{rms} metric to specify or compare the performance of RS machines it is very important that you understand how the measurement method, and specifically the cutoff frequency and input filtering, can affect the resultant calculation. Any use of the G_{rms} metric should include a clear description of these parameters to allow it to be interpreted correctly.

If you examine the Power Spectrum display and consider that G_{rms} is proportional to the area under that curve, you can immediately see that the cutoff frequency used for the G_{rms} calculation can greatly affect the value calculated. With a broad band signal such as is generated from a repetitive shock vibration system, the difference between a calculation based on a 2.5 kHz cutoff frequency and a 5 kHz cutoff frequency can be very dramatic (figure 5j). Comparing two systems based only on the G_{rms} calculated to 2.5 kHz can result in erroneous conclusions.

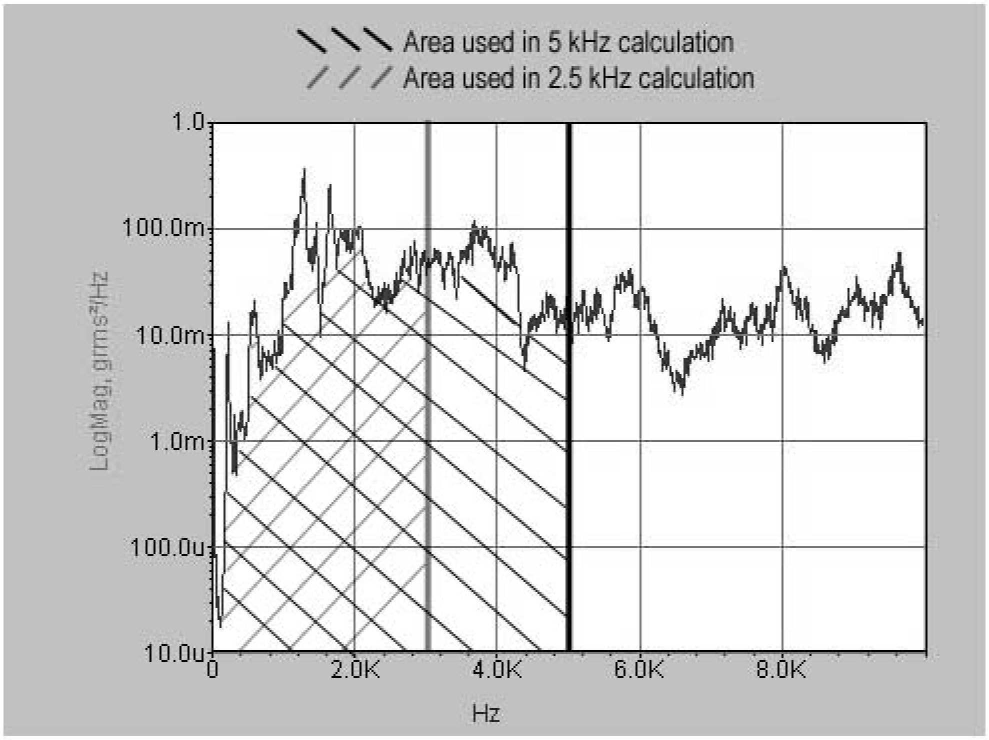


Figure 5. rms calculation using two different cutoff frequencies.

Similarly, a specification of G_{rms} in specific frequency bands can provide a method to more accurately specify a desired spectral content in a system. Some spectrum analyzers allow this calculation only over certain ranges. However, it is not difficult to calculate G_{rms} over any desired range using stored data from the spectrum analyzer and a spreadsheet program.

Another source of variation in G_{rms} calculations can arise when rms converter circuitry is used. The analog input filter on the circuit affects the value in a way similar to the effects of cutoff frequency in the frequency domain measurement. By filtering the input you are limiting the frequency range of the signal that is used in the rms measurement. Just like in the frequency domain example above, a measurement done with rms converter circuitry with a 2 kHz input filter will be very different from one done with a 5 kHz input filter.

Even more variables are introduced when you try to compare the G_{rms} values from an analog rms converter circuit to those obtained from a spectrum analyzer. A typical rms converter circuit might have an input filter set at 5kHz. However, this filter may be a single or two pole filter with the "cutoff" being the –3dB point of the filter. This means that, while the energy beyond 5 kHz has been attenuated as described by the roll-off of the filter, it has not been immediately reduced to zero past 5 kHz, as is the case with the digital cutoff of the spectrum analyzer. Also, the analog filter in the rms circuitry attenuates the signal some before the 5kHz 3 dB point as well. Consequently, the G_{rms} value provided by the rms converter can be very different from the value provided by a spectrum analyzer even though they specify the same cutoff frequency.

These differences can vary based on the signal content. In the example being considered here, the presence or absence of a peak in the power spectrum at 5500 Hz would have no effect on the spectrum analyzer G_{rms} value due to the sharp digital cutoff at 5000 Hz. However, if rms converter circuitry is used, some of the energy at 5500 Hz would still be included in the G_{rms} measurement because it would not be completely attenuated by the filter.

Conclusion

It has been said that G_{rms} is one of marketing's favorite specifications – you can make it whatever you want, just by choosing the cutoff frequencies correctly! This isn't too far from the truth. The calculation can be made from both time domain and frequency domain data. Each method has it's own set of variables that can affect the calculation. There is no industry standard or preferred method for doing the calculation and different methods are used in different vibration systems. Before comparing G_{rms} values from different machines it is important to know how the measurement was made. If you are using G_{rms} as a test specification, be sure to specify the frequency range over which the measurement is taken, and filter specifications as necessary. This will help insure the accurate reproduction of your desired tests.

References

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[1] Analog Devices, Application Note AN-256, "RMS to DC converters ease measurement tasks"
[2] Steinberg, Dave S., V*ibration Analysis for Electronic Equipment*, John Wiley & Sons, 1988
[3] Brighan, E. Orna, *The Fast Fourier Transform*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1974